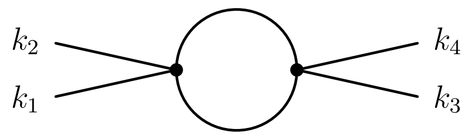
Homework #6

Please hand in by next Thursday (November 27), either during the lecture or email to <Philipp.Kleinert@physics.ox.ac.uk>. Tutorials location: Seminar Room (21. November), Conference Room (28. November).

In this homework we compute the diagram



in 4-d and dimensional regularization.

1. Write the coupling constant as λ_0 times a suitable power of the mass scale M in $4 - \omega$ dimensions.

2. Show that the Feynman rules in momentum space and after Wick rotation yield

$$A(\omega) = \frac{\lambda_0^2}{2} M^{2\omega} \int \frac{d^{4-\omega}\ell}{(2\pi)^{4-\omega}} \frac{1}{\ell^2 + m^2} \frac{1}{(\ell - k_1 - k_2)^2 + m^2}$$

   for the amputated diagram.

3. Use Feynman's identity

$$\frac{1}{a_1 \cdots a_n} = \frac{1}{(n-1)!} \int_{\{x_j \geq 0\}} \frac{\left(\prod dx_j\right)\, \delta\left(\sum x_j - 1\right)}{(a_1 x_1 + \cdots + a_n x_n)^n}$$

   to rewrite the loop integration as

$$A(\omega) = \frac{\lambda_0^2}{2} M^{2\omega} \int_0^1 dx \int \frac{d^{4-\omega}p}{(2\pi)^{4-\omega}} \frac{1}{\left(p^2 + x(1-x)(k_1+k_2)^2 + m^2\right)^2}$$

4. Show that the momentum integral evaluates to

$$A(\omega) = M^{\omega} \frac{\lambda_0^2}{32\pi^2} \left[\frac{2}{\omega} - \gamma - \int_0^1 dx\ \ln\left(\frac{m^2 + (k_1+k_2)^2 x(1-x)}{4\pi M^2}\right) + O(\omega)\right]$$

5. Unlike the tadpole, the finite part of $A(\omega)$ depends on the external momenta. In the four-point function, this diagram appears three times with permutations of $k_1, \ldots, k_4$. Draw the three graphs. Is their ω-independent part of the sum $A(\omega; k_1, k_2) + A(\omega; k_1, k_3) + A(\omega; k_1, k_4)$ an analytic function of the external momenta? Hint: We are still in Euclidean space.