Homework #4

Please hand in by next Thursday (November 13), either during the lecture or email to <Philipp.Kleinert@physics.ox.ac.uk>. Tutorials location: Seminar Room (7. November), Fisher Room (14. November).

1. The specific heat $C = \frac{\partial \langle E \rangle}{\partial T}$ measures how much heat is required to affect a temperature change. Show that it satisfies

$$C = \frac{1}{k_B T^2} \left[\langle E^2 \rangle - \big(\langle E \rangle \big)^2 \right]. \tag{1}$$

2. Produce a plot that shows the specific heat as it crosses the phase transition line. You can find instructions for the lattice simulation code at `http://vbraun.cc/qft/?page_id=116`. Either install Sage on your own computer / lab, or run it at `http://www.sagemathcloud.com`. Note: There is already a `Simulation.specific_heat` observable implemented, don't reinvent the wheel.

3. Another observable is the Binder cumulant (`Simulation.Binder_cumulant`):

$$U = 1 - \frac{\langle \overline{\phi}^4 \rangle}{3 \big(\langle \overline{\phi}^2 \rangle \big)^2}. \tag{2}$$

   Which values does it take far in the broken / far in the unbroken phase? Confirm your answer with a numeric calculation.

Installation:

```
$ git clone https://github.com/vbraun/lattice_phi4.git
$ sage -sh -c make
```

Running a simple simulation ($\mu_L^2 = -1.26$, $\lambda_L = 1.0$):

```
sage: from phi4 import Simulation
sage: sim = Simulation(-1.26, 1.0, equilibrate=1000)
sage: sim.run(1000)    # run for 1000 iterations
sage: sim.abs_phi().samples()
sage: sim.abs_phi().average()
```

Plotting:

```
sage: from phase_plot import PhasePlot
sage: susz = PhasePlot(32, 32,
....:    observable=Simulation.susceptibility,
....:    equilibrate=1000, iterations=1000)
sage: susz.mass_squared = srange(-2, 0, 0.05)
sage: susz.plot()
```