Homework #3

Please hand in by next Thursday (November 6), either during the lecture or email to `<Philipp.Kleinert@physics.ox.ac.uk>`. Tutorials location: Conference Room (31. October), Seminar Room (7. November).

1. Verify that the Feynman propagator is a Green's function by acting on it with the Klein-Gordon operator $-\partial_x^2 + m^2$.

2. Let ψ be a complex-valued[1] field with Lagrangian density
$$\mathcal{L} = -\partial_\mu \psi \partial^\mu \psi^* - |\psi|^2. \tag{1}$$
Show that $\mathcal{L}$ is real and invariant under the symmetry $\psi \mapsto e^{i\theta}\psi$ for a constant $\theta \in \mathbb{R}$.

3. Add sources $\mathcal{L} \mapsto \mathcal{L} + J^*\psi + J\psi^*$ to the Lagrangian and evaluate the path integral $Z(J, J^*)$.

4. Compute the time-ordered two-point functions
$$\langle 0|T\psi(x)\psi(y)|0\rangle, \quad \langle 0|T\psi(x)\psi^*(y)|0\rangle, \quad \langle 0|T\psi^*(x)\psi^*(y)|0\rangle. \tag{2}$$

5. Use Wick's theorem to evaluate the time ordering of the product of two normal ordered squares
$$T\Big(:\phi(x)^2: \;\times\; :\phi(y)^2: \Big). \tag{3}$$
Show that this is finite, that is, can be written in terms of normal ordered products and propagators *not* evaluated at zero. Hint: Write it as particular limit of 4 distinct points. You only need to apply Wick's theorem repeatedly.

[1] So far, we only used real-valued fields in the lecture.