Homework #1

Please hand in by next Thursday (October 23)

1. The position-space creation and annihilation operators satisfy

$$\begin{aligned}[a(\vec{x}), a(\vec{y})] = [a^\dagger(\vec{x}), a^\dagger(\vec{y})] = 0 \\ [a(\vec{x}), a^\dagger(\vec{y})] = \delta^3\big(\vec{x} - \vec{y}\big)\end{aligned} \tag{1}$$

   Show that the position-space Hamiltonian

$$\begin{aligned} H = \int d^3x\; a^\dagger(\vec{x}) \left(-\frac{1}{2m} \sum_i \frac{\partial^2}{\partial x_i^2} + U(\vec{x}) \right) a(\vec{x}) \\ + \frac{1}{2} \int d^3x\; d^3y\; a^\dagger(\vec{x}) a^\dagger(\vec{y}) V(\vec{x} - \vec{y}) a(\vec{x}) a(\vec{y}) \end{aligned} \tag{2}$$

   preserves the total number of particles measured by

$$N = \int d^3x\; a^\dagger(x) a(x). \tag{3}$$

2. Apply the abstract Schrödinger equation $i\partial_t|\psi, t\rangle = H|\psi, t\rangle$ to the n-particle state

$$|\psi, t\rangle = \int d^3x_1\; d^3x_2 \cdots d^3x_n\; \psi(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n; t)\; a^\dagger(\vec{x}_1) \cdots a^\dagger(\vec{x}_n)|0\rangle \tag{4}$$

   with the Hamiltonian eq. (2). Conclude that the position-space wave function satisfies

$$\begin{aligned} i\frac{\partial}{\partial t}\psi(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n; t) = \Bigg[\sum_{j=1}^{n} \Big(-\frac{1}{2m} \sum_i \frac{\partial^2}{\partial (\vec{x}_j)_i^2} + U(\vec{x}_j) \Big) \\ + \frac{1}{2} \sum_{j,k=1}^{n} V(\vec{x}_j - \vec{x}_k) \Bigg] \psi(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n; t). \end{aligned} \tag{5}$$

3. Show that $|\psi, t\rangle$ is invariant under exchange of any two particles.