Homework #7

Please hand in by next Thursday (December 4), either during the lecture or email to <Philipp.Kleinert@physics.ox.ac.uk>. Tutorials location: Conference Room (28. November), Fisher Room (5. December). There is no tutorial for this final homework.

Consider the massless ϕ^4 theory with Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4!}\lambda\phi^4 \tag{1}$$

In the lecture we derived the Coleman-Weinberg effective potential[1]

$$V_{\text{eff}}(\varphi) = V(\varphi) - \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\log\left(\frac{k^2 - V''(\varphi)}{k^2}\right) \tag{2}$$

1. Classify all 1-loop diagrams in ϕ^4 theory (with an arbitrary number of external legs).
2. The 1-loop contribution to the potential (after Wick rotation)

$$V_{\text{1-loop}}(\varphi) = \int \frac{d^4k}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{1}{2n}\left(\frac{\frac{1}{2}\lambda\varphi^2}{k^2}\right)^n \tag{3}$$

   comes from the Feynman diagrams where the constant mean field φ is inserted at each external leg. Explain the symmetry factors $\frac{1}{2n}$ and $\frac{1}{2}$ in the sum.
3. Check that the one-loop result matches the Coleman-Weinberg effective potential, that is, $V_{\text{eff}} = V + V_{\text{1-loop}}$.

[1]All formula on this sheet are in 4-d and after Wick rotation.